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ABSTRACT

Commonality analysis is an attempt to understand the relative predictive power of the regressor variables, both individually and in combination. The squared multiple correlation is broken up into elements assigned to each individual regressor and to each possible combination of regressors. The elements have the property that the appropriate sums not only add to squared multiple correlations with all regressors, but also to the squared multiple correlation of any subset of variables, including the simple correlations. Commonality analysis may be used as a procedure to guide a stepwise regression. Commonality analysis does not tell us anything that cannot be deduced from a table of squared multiple correlations. However, commonality analysis does help us make comparisons in an organized manner. The purpose of this paper is to explore commonality procedures, to develop its properties, and to present a multivariate generalization for the explorations of commonality in a situation where there is more than one regressor. A new computer-oriented algorithm is also presented. (Author/BJG)

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COMMONALITY

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March 2, 1973

1.0 Introduction

Regression analysis is often used in both the physical and social sciences. However, regression analysis is most useful when the so-called independent variables are truly independent in the sense that they can be experimentally manipulated, e.g., an experimenter can fix the values of all but one independent variable, then vary the value of the remaining variable; then choose a second variable to manipulate, and so forth. The principles of experimental design show that for a given sample size and assuming an underlying linear relationship the experimenter should select values of the independent variables to be far apart and uncorrelated, resulting in unbiased estimates of the regression coefficients that are most precise ceteris paribus in the sense that their standard errors are smallest. In this case, the squared multiple correlation is simply the sum of the squares of the simple correlations between the independent and dependent variables.

In survey research, random samples of persons are selected from a defined population. Most variables cannot be independently manipulated as in a designed experiment; although stratified sampling may avoid intercorrelation among some regressors, it is very difficult to stratify on many variables. Unbiased estimates

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of parameters can be computed from such samples, but the precision of the estimates is lessened by correlations among regressor variables. Furthermore, the regressor variables do not contribute independently to the squared multiple correlation, for the squared multiple correlation may be either larger or smaller than the sum of the squared simple correlations. Regression analysis may become more a tool for measuring correlation or predictive power than for estimating parameters of a causal system.

Commonality analysis is an attempt to understand the relative predictive power of the regressor variables, both individually and in combination. The squared multiple correlation is broken up into elements assigned to each individual regressor and to each possible combination of regressors. The elements have the property that the appropriate sums not only add to squared multiple correlations with all regressors, but also to the squared multiple correlation of any subset of variables, including the simple correlations. Commonality analysis may be used as a procedure to guide a stepwise regression.

Commonality analysis does not tell us anything that cannot be deduced from a table of squared multiple correlations. However, commonality analysis does help us make comparisons in an organized manner.

Commonality analysis is not new. Kempthorne (1957, p. 304ff) suggests the procedure briefly. Creager and Valentine (1962) credit Bottenberg and Ward (1963) for the same procedure although with a different focus. Newton and Spurrell (1967a, 1967b) arrived at the

same method and present interpreted examples. The name commonality was suggested by Mood (1971) who derived the solution independently. A generalization for sets of variables was reported by Wisler (1969) who also showed the relationship between commonality analysis and part correlations. Creager (1971a) has commented on the procedure and compared it to a factor analytic procedure (1971b). Mayeske et al (1969, 1973) have used the technique extensively.

The purpose of this paper is to explore the procedure further, to develop its properties, and to present a multivariate generalization for the explorations of commonality in a situation where there is more than one regressand. A new computer-oriented algorithm is also presented.

2.0 Univariate Commonality

Univariate regression analysis assumes a linear model

$$\underset{\sim}{y} = \underset{\sim}{X}\underset{\sim}{\beta} + \underset{\sim}{\varepsilon}$$

where y is a column vector with elements y_i ($i = 1,2,\ldots,N$) representing the observed values of the regressand (also called dependent variables or criterion), $\underset{\sim}{X}$ is an N x m matrix with elements x_{ij} ($j = 1,2,\ldots,m$) representing the observed values of the m regressor (also called independent or predictor) variables for the N observations, $\underset{\sim}{\beta}$ is a column vector of m unknown regression coefficients, and $\underset{\sim}{\varepsilon}$ is a column vector with elements ε_i representing the unknown residuals (also called errors). For simplicity, $\underset{\sim}{y}$ is assumed to have a zero mean although all equations can be modified to allow a non-zero value; X is of rank m, the ε_i are assumed $NID(0,\sigma^2)$.

Commonality analysis requires the definition of the regression equations between $\underset{\sim}{y}$ and each possible combination of the m variables in $\underset{\sim}{X}$. To identify these regressions we define a submodel

$$\underset{\sim}{y} = \underset{\sim s}{X}\underset{\sim s}{\beta} + \underset{\sim s}{\varepsilon}$$

where the subscript s represents in ascending order the n_s indices of the columns of $\underset{\sim}{X}$ contained in the model. We also define the complementary subscript $\bar{s}$ which represents the $n_{\bar{s}} = m - n_s$ indices in ascending order of all columns of X not included in s. For example, if $\underset{\sim}{X}$ has 6 variables and s = (13), then $\underset{\sim s}{X}$ is an N by 2 matrix containing the first and third columns of $\underset{\sim}{X}$, $\bar{s}$ = (2456), and $\underset{\sim \bar{s}}{X}$ is an N by 4 matrix containing the second, fourth, fifth and sixth

columns of $\underset{\sim}{X}$. s may be dropped if it contains all m subscripts in which case $\bar{s}$ is the null set. y does not require a subscript. $\underset{\sim}{\varepsilon}_s$ is NID$(0,\sigma_s^2)$ where

$$\hat{\varepsilon}_s'\hat{\varepsilon}_s = \hat{\varepsilon}'\hat{\varepsilon} + \underset{\sim}{\beta}_{\bar{s}\cdot s}'(\underset{\sim}{X}_{\bar{s}}'\underset{\sim}{X}_{\bar{s}} - \underset{\sim}{X}_{\bar{s}}'\underset{\sim}{X}_s(\underset{\sim}{X}_s'\underset{\sim}{X}_s)^{-1}\underset{\sim}{X}_s'\underset{\sim}{X}_{\bar{s}})\underset{\sim}{\beta}_{\bar{s}\cdot s}$$

and $\underset{\sim}{\beta}_{\bar{s}\cdot s}$ is the regression coefficients corresponding to the $n_{\bar{s}}$ variables not included in s and $\hat{\varepsilon}$ is the estimate of ε computed using all variables in the model.

The statistics usually computed in a regression analysis are estimates of the regression coefficients

$$\hat{\beta}_s = (X_s'X_s)^{-1}X_s'y \ ;$$

predicted values of the regressand,

$$\hat{y}_s = X_s\hat{\beta}_s \ ;$$

the residual vector

$$\hat{\varepsilon}_s = y - X_s\hat{\beta}_s \ ;$$

the residual variance

$$\hat{\sigma}_s^2 = \frac{1}{N-n_s-1}\hat{\varepsilon}_s'\hat{\varepsilon}_s \ ;$$

the covariance of β_s

$$\hat{\Sigma}_{\beta s} = \frac{\hat{\sigma}_s^2}{N-n_s-1}(X_s'X_s)^{-1} \ ;$$

and of the squared multiple correlation (SMC)

$$\hat{R}^2_{ys} = \frac{\hat{y}_s' \hat{y}_s}{y'y}$$

The $\hat{R}^2_{ys}$ will ordinarily be written without the circumflex. Simple (Pearson product-moment) correlations may be written as r_{ij} for the correlation of columns X_i and X_j of X and as r_{yj} for the correlation of Y with X_j.

The estimates of parameters for a subset s are not in general the same estimates of the parameters in the model as those in which all regressors are included. For example,

$$\hat{\beta}_s = \hat{\beta}_{s.\bar{s}} - (X_s'X_s)^{-1} X_s'X_{\bar{s}}\hat{\beta}_{\bar{s}.s}$$

which indicates $\hat{\beta}_s$ is the same as $\beta_{s.\bar{s}}$ only if $X_s'X_{\bar{s}} = 0$ or $\beta_{\bar{s}.s} = 0$, that is, if X_s and $X_{\bar{s}}$ are uncorrelated or the regression coefficients of $X_{\bar{s}}$ are zero given X_s.

The variance of the regressand y may be partitioned

$$\hat{\sigma}_y^2 = \hat{\beta}_1^2\hat{\sigma}_1^2 + \hat{\beta}_2^2\hat{\sigma}_2^2 + \ldots + \hat{\beta}_m^2\hat{\sigma}_m^2 + 2\sum_{i=1}^{m}\sum_{j=i+1}^{m}\hat{\beta}_i\hat{\beta}_j\widehat{cov}_{ij} + \hat{\sigma}_e^2$$

where the $\hat{\sigma}_i^2$'s are the estimated variances of the x's, the $\widehat{cov}_{ij}$'s are their covariances, and $\hat{\sigma}_e^2$ is the unpredictable variance. The variance of y, therefore, can be broken into two parts

$$\hat{\sigma}_y^2 = \hat{\sigma}_{\hat{y}}^2 + \hat{\sigma}_e^2$$

where $\hat{\sigma}_{\hat{y}}^2$ is the part of the variance of y associated with and predictable from the independent variables.

Newton and Spurrell (1967a) discuss two facets of multiple regression analysis: prediction and operation. These two uses complement each other since an understanding of a process can lead to more efficient prediction and prediction is often the measure of understanding of a process. In both cases one may wish to remove variables of little importance from a set of available variables. This is, of course, one of the purposes of the analysis of variance which is so often used to test hypotheses that one or more regression coefficients are not appreciably different from zero, a hypothesis equivalent to testing whether or not some independent variables are of little predictive importance.

If the independent variables are mutually uncorrelated, then the standard analysis of variance is quite useful, for we can see which, if any, of the independent variables contribute significantly

to prediction. If the x's are correlated, however, an orthogonal anal
of variance requires an a priori ordering of the variables as in

$$\hat{\sigma}^2_y = \beta^2_1 \hat{\sigma}^2_1 + \beta^2_{2.1} \hat{\sigma}^2_{2.1} + \beta^2_{3.12} \hat{\sigma}^2_{3.12} + \ldots + \beta^2_{m.12\ldots m-1} \hat{\sigma}^2_{m.12\ldots m-}$$

where the β's and $\hat{\sigma}$'s are partial regression coefficients and variances, respectively; that is, $\beta^2_{j.12\ldots j-1} \hat{\sigma}^2_{j.12\ldots j-1}$

represents the regression coefficient and variance of variable i with variables 1 through j-1 partialed out. This procedure is useful if there is an a priori reason for ordering these independent variables, but there are m! different ways that the variables may be ordered and different orderings will usually affect the measured contribution to the predicted sum of squares. The problem is that any variance in common between two variables is allocated entirely to the former of the two variables in an ordering; any predictive power a third variable has in common with two in an earlier position is allocated entirely to the first two, and so forth. The ordering, therefore, may indeed affect our estimate of the importance of independent variables.

The purpose of commonality analysis is to partition a squared multiple correlation into elements associated with each (regressor) variable and into elements associated with each possible combination of regressors. The analysis shows in some sense to what extent each individual variable affects the SMC correlation by itself and how much it affects the SMC in combination with other variables. If two variables have no commonality, then the contribution of either is not affected by the entrance of the other variable into the regression equation. If two variables have a non-zero commonality, then the entry of either variable into a regression equation will affect the contribution of the other.

Commonality analysis generates elements such that the sum of all elements equals the squared multiple correlation. It is also required that the sum of all elements associated with a single variable total to the squared simple correlation of that variable with a regressand, that the sum of all elements associated with either or both of two variables total to the squared multiple correlation of those two variables with the regressand, and so forth.

These relationships can be expressed for the two-regressor (m=2) case as

$$R^2_{y12} = U_1 + U_2 + C_{12} \qquad (1)$$

where U_1 is the "uniqueness" or "unique" contribution of X_1 to the SMC, U_2 is the "unique" contribution of X_2, and C_{12} is the common element or commonality. The uniquenesses are considered first

order commonalities. The contribution of X_1 alone is

$$r^2_{y1} = U_1 + C_{12} \tag{2}$$

and the contribution of X_2 alone is

$$r^2_{y2} = U_2 + C_{12} \cdot \tag{3}$$

The relationships in equations 1, 2, and 3 can be written in matrix form as

$$\underset{\sim}{R} = \underset{\sim}{G}\underset{\sim}{C} \tag{4}$$

where

$$\underset{\sim}{r} = \begin{bmatrix} R^2_{y(1)} \\ R^2_{y(2)} \\ R^2_{y(12)} \end{bmatrix}, \quad \underset{\sim}{G} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \quad \underset{\sim}{C} = \begin{bmatrix} U_1 \\ U_2 \\ C_{12} \end{bmatrix} \tag{5}$$

We can then solve equation 5 for the commonality by

$$\underset{\sim}{C} = G^{-1}\underset{\sim}{r} \tag{6}$$

in which

$$G^{-1} = \begin{bmatrix} 0 & -1 & 1 \\ -1 & 0 & 1 \\ 1 & 1 & -1 \end{bmatrix}$$

and the explicit solution for C is

$$U_1 = R^2_{y12} - R^2_{y2} \tag{7}$$

$$U_2 = R^2_{y12} - R^2_{y1} \tag{8}$$

$$C_{12} = R^2_{y1} + R^2_{y2} - R^2_{y12} \tag{9}$$

U_1 and U_2 are in fact sums of squares and as such must be non-negative. C_{12} may be either positive, negative, or zero.

The logic for the three-variable case is similar. There are 2^3-1 possible combinations of variables and, therefore, 7 possible SMC's. The definition of the SMC's in terms of the commonalities are

$$\begin{bmatrix} R_1^2 \\ R_2^2 \\ R_3^2 \\ R_{12}^2 \\ R_{13}^2 \\ R_{23}^2 \\ R_{123}^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ C_{12} \\ C_{13} \\ C_{23} \\ C_{123} \end{bmatrix}$$

solving for the commonalities, we find

$$\begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ C_{12} \\ C_{13} \\ C_{23} \\ C_{123} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 0 & -1 & 0 & 1 \\ 0 & 0 & 0 & -1 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 & 1 & 1 & -1 \\ 0 & -1 & 0 & 1 & 0 & 1 & -1 \\ -1 & 0 & 0 & 1 & 1 & 0 & -1 \\ 1 & 1 & 1 & -1 & -1 & -1 & 1 \end{bmatrix} \begin{bmatrix} R_1^2 \\ R_2^2 \\ R_3^2 \\ R_{12}^2 \\ R_{13}^2 \\ R_{23}^2 \\ R_{123}^2 \end{bmatrix}$$

As the commonality vector becomes more complex, it is useful to organize the elements into a commonality table such as shown in Figure 1

(INSERT FIGURE 1 ABOUT HERE)

The margin on the right-hand side contains the vector of commonality coefficients and sums to $R^2_{y12...m}$. The columns in the body of the table include the elements of each regressor and thus these columns sum to the squared simple correlations.

Some authors (e.g. Mayeske et al,1969) have found the commonality table more useful when unitized, that is, when all elements are divided by the SMC. In this case, the element in the lower right-hand corner is unity and the elements add to unity instead of to the SMC. Unitized elements can be thought of as proportions (percents) of predictable variance instead of proportions (percents) of total variance.

The generalization of commonality to four or more independent variables is straightforward. If there are m variables, then we have $M = 2^m-1$ equations in M unknowns formed by defining each possible SMC as the sum of all elements common to its regressors. The formation and solution of such equations become tedious if M is large, but computer algorithms such as the one presented in Section 7 use a much simpler procedure and avoid the solution of large sets of equations. The size of the commonality table may also be reduced through definition of sets of variables which are treated as a unit; this procedure is discussed in Section 5.

Figure 1

3-Variable Commonality Table

X_1	X_2	X_3	$\underline{C}$
U_1			U_1
	U_2		U_2
		U_3	U_3
C_{12}	C_{12}		C_{12}
C_{13}		C_{13}	C_{13}
	C_{23}	C_{23}	C_{23}
C_{123}	C_{123}	C_{123}	C_{123}
$R^2_{Y(1)}$	$R^2_{Y(2)}$	$R^2_{Y(3)}$	$R^2_{Y(123)}$

The partitioning of the SMC is, then,

$$R^2_{ys} = \sum_{i=1}^{n_s} C_i + \sum_{i=1}^{n_s} \sum_{j=i+1}^{n_s} C_{ij} + \sum_{i=1}^{n_s} \sum_{j=i+1}^{n_s} \sum_{k=j+1}^{n_s} C_{ijk} + \ldots$$

Wisler (1969) has expressed commonalities in terms of part correlations which have been discussed by DuBois(1957). A part correlation is the correlation between two variables with one of them measured as a residual from one or more other variables. (Partial correlations measure the variables as residuals). The part correlation

$$r_{y(1.2)} = \frac{r_{01} - r_{02}r_{12}}{\sqrt{1-r^2_{12}}}$$

is the correlation of y with $X_{1.2}$ where $X_{1.2}$ is the residual of X_1 from the linear regression of X_1 on X_2. Wisler has shown that uniqueness for any variable X_j is the squared part correlation of Y with X_j when all other variables are regressed out of X_j. The commonalities can then be expressed in terms of these squared part correlations.

Since commonality analysis may be used for deciding which regressor variables to include in a regression equation, it is a procedure for stepwise regression analysis. If the research aim is solely maximization of prediction for a given number of variables, then commonality analysis is unnecessary since all possible

SMC's are computed as part of a commonality analysis. Stepwise regression may proceed as follows:

Using Newton and Spurrell (1967b) Rule 1 which maximizes the reduction in residual sum of squares at each stage of the regression by adding the variable that reduces the SMC the most, we would simply add the variable with the highest sum of commonalities We would then draw a vertical line through that column and a horizontal line through each row which contains a commonality in that column. We would then sum the columns again, resulting in new totals that would correspond to the covariance of the three remaining variables with the largest partialed out. After selecting a second variable, we would cross out the associated rows and columns, retotal and proceed. This procedure corresponds to adding the variable with the largest partialed correlation.

Newton and Spurrell suggest three other rules, including (2) selecting the variable with the largest primary (unique) element, (3) selecting variables in which the uniqueness is large compared with any related secondary element and (4) selecting only one variable included in the large positive secondary element.

Significance tests are not known for all commonalities, but the uniqueness U_j is the contribution of X_j to the SMC after all other regressors are included in the regressor equation. The test for a significant additional contribution to the SMC is

$$F^1_{N-m-1} = \frac{U_j}{1-R^2_{y12\ldots m}} \quad (N-m-1)$$

the square root of which is distributed as t with N-m-1 degrees of freedom. This significance test is equivalent to testing that the partial regression coefficient $\beta_j = 0$. An insignificant F or t indicates that the contribution to $R^2_{y12\ldots m}$ of X_j is consistent with the hypothesis that β_j is zero and the value U_j is due to sampling error. The individual tests of many partial regression coefficients are commonly used although the tests are not strictly independent.

The significance of the contribution of several or all variables to the SMC may be calculated by summing the unique contributions of the several variables and their common elements. For example, if m = 4, the test of the contribution of X_3 and X_4 would be

$$F^2_{N-S} = \frac{C^*}{1-SMC} \cdot \frac{N-S}{2}$$

where C* is the sum of the elements that would be excluded from the SMC if the K regressors were dropped. This test is equivalent to testing that all K β_j's are simultaneously zero.

Although there is no known direct test of the significance of a commonality, we may judge their size under certain circumstances. For example, if m = 4 and we are interested in the magnitude of C_{34}, then we might test the hypotheses

$$H_3 : U_3 = 0$$

$$H_4 : U_4 = 0$$

$$H_{34} : U_3 + U_4 + C_{34} = 0$$

as shown above. If all three hypotheses are accepted, then C_{34} is not inconsistent with a population value of zero. If H_3 and H_4 are accepted, but H_{34} rejected, then we may conclude the C_{34} is non-zero. If, however, H_3 or H_4 is not accepted, then H_{34} does not isolate C_{34} and any inferences are dubious. This procedure can be used recursively for more complex commonalities.

This testing procedure has severe inferential problems even though examples where H_3 and H_4 are accepted while H_{34} are rejected are easy to construct. The problem is that if the actual population values of the uniquenesses of U_3 and U_4 were precisely zero, then the value C_{34} must also be zero, unless X_3 and X_4 are perfectly correlated in which case C_{34} is indeterminate. It is, therefore, problematic to accept zero values for U_3 and U_4, then continue to question the size of C_{34}.

3.0 Numerical Example

The purpose of this section is to show what a commonality table looks like and to give some hints as to how one might look at a table. Computational procedures will be discussed in section 7.

Figure 3-1 displays all possible squared multiple correlations between one regressand and four regressors. These five variables were collected on a sample of eighty students.

(INSERT FIGURE 3-1 ABOUT HERE)

Figure 3-2 displays a commonality table. The SMC with all regressors entered is .6825 which indicates that over 68% of the sum of squares of Y can be explained by these four variables. The F statistic for this SMC is 40.30 (ndf=4,75) which is highly significant. The simple squared correlations range from .3967 to .6351 and are individually highly significant when subjected to the ordinary test for the significance of a correlation. We conclude, therefore,

Figure 3-1

Squared Multiple Correlations (r)
for all combinations of 4 predictors

		s
1	0.3967	1
2	0.4643	2
3	0.5378	12
4	0.5269	3
5	0.6080	13
6	0.5783	23
7	0.6245	123
8	0.6351	4
9	0.6684	14
10	0.6443	24
11	0.6707	124
12	0.6528	34
13	0.6821	134
14	0.6571	234
15	0.6826	1234

that there is some predictable variance worth looking at.

(INSERT FIGURE 3-2 ABOUT HERE)

Figure 3-2

TEST OF COMMONALITY PROGRAM

FOUR VARIABLE PROBLEM

THE DEPENDENT VARIABLE IS

SET ACRONYM	SET 1	SET 2	SET 3	SET 4
FIRST ORDER				
U(X)	0.0255	0.0005	0.0119	0.0581
SECOND ORDER				
C 12	0.0038	0.0038		
C 13	0.0009		0.0009	
C 14	0.0207			0.0207
C 23		0.0018	0.0018	
C 24		0.0160		0.0160
C 34			0.0747	0.0747
THIRD ORDER				
C 123	0.0030	0.0030	0.0030	
C 124	0.0311	0.0311		0.0311
C 134	0.0264		0.0264	0.0264
C 234		0.1228	0.1228	0.1228
FOURTH ORDER				
C 1234	0.2853	0.2853	0.2853	0.2853
R2(X)(squared simple correlation)	0.3967	0.4643	0.5269	0.5351
R2(TOT)(squared multiple correlation)	0.6826	0.6826	0.6826	0.6826
PCNT	0.0374	0.0007	0.0174	0.0852

$F^4_{75} = 40.30$

X_1: $t_{75} = 2.45$

X_2: $t_{75} = .34$

X_3: $t_{75} = 1.68$

X_4: $t_{75} = 3.70$

There are no negative commonalities in this example. Negative commonalities are possible but not common in educational data. A negative commonality indicates that one variable actually confounds the predictive power of another. A hypothetical example may explain this phenomenon. Both weight and speed are important to success as a professional football player and each would be moderately correlated with a measure of success in football. Weight and speed are presumably negatively correlated and would have a negative commonality in predicting success in football. If both weight and speed are known, one would expect to make a much better prediction of success using both variables to select fast, heavy men rather than just selecting the fastest regardless of weight or heaviest regardless of speed. Thus the negative commonality indicates that explanatory power of either is greater when the other is also used.

The uniquenesses are shown at the top of the figure 3-2 and the t statistics at the bottom. The uniquenesses indicate the amount of variance explained by each variable after all other variables are entered into the equation. X_1 and X_4 have significant large uniquenesses and t statistics. X_2 does not look worth keeping, and X_3 could be a sampling fluctuation and seems to add little.

Since one and perhaps two variables add little, we ask where the original predictive power of these variables went. The commonality C_{1234} indicates that 28% of the explained sum of squares is common to the four variables, that is, entering any one of the four variables into the equation will increase the SMC by at least .28.

Clearly, much of the explanatory power of these four variables is redundant.

The commonality C_{234} is fairly large indicating that the use of any of these variables will explain some of the same predictable variance as the others. The next largest value is C_{34} which shows the common element of X_3 and X_4.

To summarize, the table seems to indicate that X_2,X_3, and X_4 predict much the same part of the explainable sum of squares, in fact, the predictive power of X_2 is almost completely common to the others. Since X_4 is most powerful and X_1 is most different from X_4, we may expect that the SMC R^2_{y14} is quite large. Figure 3-1 shows us that this SMC is .6684 or just .0142 less than the SMC with all four variables.

4.0 Two-Variable Commonality

Commonality is a complex measure so it may be instructive to look carefully at the simple case of two regressor variables.

The squared multiple correlation can be expressed as a function of three simple correlations by

$$(1) \qquad R^2_{y(12)} = \frac{r^2_{y1} + r^2_{y2} - 2r_{y1}r_{y2}r_{12}}{1 - r^2_{12}}$$

and the commonality written as

$$(2) \qquad C_{12} = r^2_{y1} + r^2_{y2} - R^2_{y\,12}$$

The commonality is, therefore, a contrast between what the SMC would have been if X_1 and X_2 were uncorrelated (i.e. $r^2_{y1} + r^2_{y2}$) and what the actual SMC is (i.e. R^2_{y12}). Substituting (1) into (2), we have

$$(3) \qquad C_{12} = \frac{2r_{y1}r_{y2}r_{12} - r^2_{y1}r^2_{12} - r^2_{y2}r^2_{12}}{1-r^2_{12}}$$

If $r_{12} = 0$, then the SMC is the sum of the squares of the two simple correlation coefficients and thus the commonality is zero and the uniqueness of U_1 is then R^2_{y1} and of U_2 is R^2_{y2}. If r_{12} does not equal zero, then the SMC may be larger or smaller than $\sqrt{r^2_{y1} + r^2_{y2}}$, depending on the sign of the product $r_{y1}r_{y2}r_{12}$ and the magnitude of r_{12}.

The complexity of the relationship between r_{12}, C_{12}, and R^2_{y12} is shown graphically in Figure 4-1. For this graph, r_{y1} and r_{y2} are considered fixed constants, .3 and .2, respectively. Given particular values of r_{y1} and r_{y2}, r_{12} cannot in general range over the entire area between -1 and +1 since extremely high or extremely low values would result in a non-positive definite correlation matrix. The boundaries for permissible r_{12} are

$$r_{Y1}r_{Y2} - \sqrt{(r_{Y1}^2-1)(r_{Y2}^2-1)} \leq r_{12} \leq r_{Y1}r_{Y2} + \sqrt{(r_{Y1}^2-1)(r_{Y2}^2-1)}$$

which for this example gives the boundary $-.8747 \leq r_{12} \leq .9947$.

Figure 4-1 graphs the values of R^2_{Y12} and C_{12} as functions of r_{12} over the permissible range. All negative values of r_{12} are associated with negative C_{12} and very high positive values also have negative commonalities. The value of C_{12} is still small at its maximum ($r_{12} = .667$) where the squared multiple correlation is at its minimum. The C_{12} curve is a (displaced) mirror image of the R^2_{Y12}.

This graph brings out the important point that the commonality may also be associated with high positive values of r_{12}.

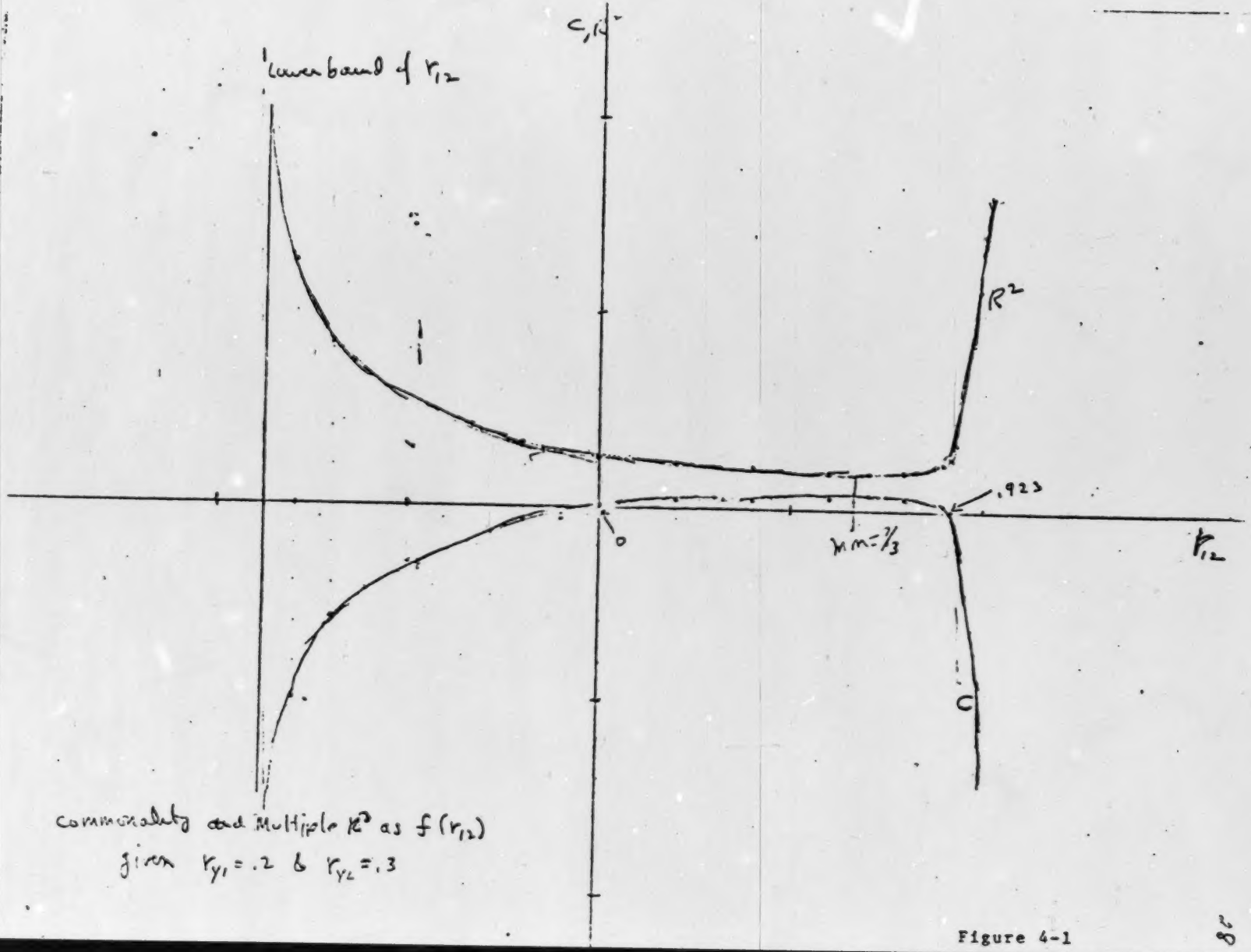
The relationship of X_1, X_2, and Y are shown graphically as in Figure 4-2. The two vectors, X_1 and X_2 are of unit length and represent variables X_1 and X_2; the vector $\hat{Y}$ represents the projection of the (unit length) vector Y on the space spanned by X_1 and X_2. The distance OA is r_{Y1}, OB is r_{Y2}, and OC is r_{12}. The length of the vector $O\hat{Y}$ is the multiple correlation R_{Y12}. The distance of $A\hat{y}$ is $U_2^{1/2}$ and the distance $B\hat{y}$ is $U_1^{1/2}$. The vector $O\hat{\hat{Y}}$ is the square root of $R^2_{y1} + R^2_{y2}$ which is what $\hat{Y}$ would be if X_1 was orthogonal to X_2 while $OA(r_{y1})$ and $OB(r_{y2})$ remained constant. (OC would be zero.) Since the commonality is the squared of $O\hat{\hat{Y}}$ minus the square of $O\hat{Y}$, $O\hat{\hat{Y}} > O\hat{Y}$ implies a negative commonality.

Figure 4-3 shows the effect of r_{12} on the multiple correlation and commonality over the range of permissible values while holding r_{y1} and r_{y2} constant. First, the terminus of any vector representing the multiple correlation must fall on the line $\hat{Y}'\hat{Y}''$ since its projection on X_1 must be orthogonal and of length OA. The

smallest possible angle (or largest cosine) of X_2OX_1 is represented by X_1OX_2' since a vector perpendicular to OX_2 must meet the vector perpendicular to OX_1 within the unit circle. The largest possible angle is X_1OX_2'' for the same reason. The actual vector X_2 must lie between X_2' and X_2''.

The dotted lines from the origin to $\hat{\hat{y}}+$ and $\hat{\hat{y}}-$ represent the two vectors where the commonality would be zero since the multiple correlation would be $\sqrt{r_{y1}^2+r_{y2}^2}$ or $\sqrt{OA^2+OB^2}$. If X_1 and X_2 are correlated highly enough, then the vector representing the multiple correlation will terminate between $\hat{\hat{y}}-$ and $\hat{y}'$ in which case the multiple correlation is larger than $\sqrt{r_{y1}^2+r_{y2}^2}$ and the commonality is negative. Also, if X_1 and X_2 are correlated lowly enough, then the terminus of the multiple correlation vector would fall between $\hat{y}+$ and $\hat{y}''$ in which case the commonality would be negative. If the vector representing the multiple correlation terminates between $\hat{\hat{y}}+$ and $\hat{\hat{y}}-$, then the vector is shorter than $\sqrt{r_{y1}^2+r_{y2}^2}$ and the commonality is positive.

These graphs show that the commonality of two variables is not a vector but the difference between the squares of two vectors and thus should not be considered a variance in itself.



communality and Multiple R^2 as $f(r_{12})$
given $r_{y1} = .2$ & $r_{y2} = .3$

Figure 4-1

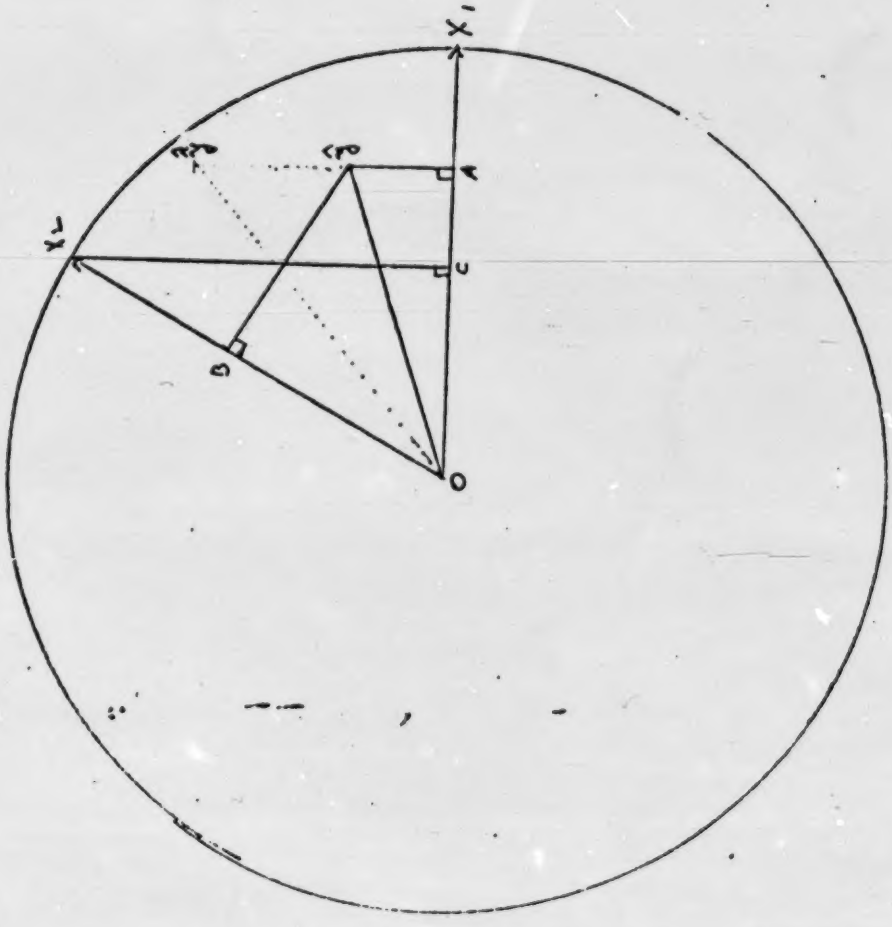


Figure 4-2

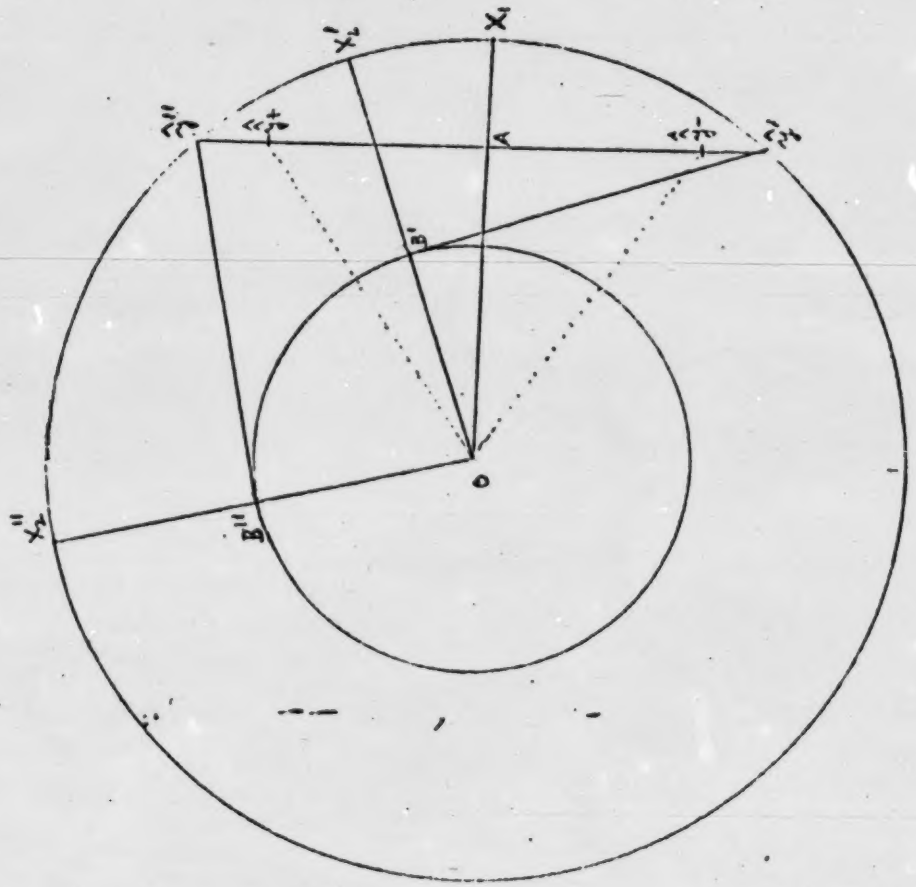


Figure 4-3

5.0 Sets of Variables

Commonality tables become unmanageably large for even a fairly modest number of regressors. For example, if there are 10 regressors the number of lines in the table is 1,024. It is therefore important to reduce the table in size.

One method is to group the regressors into logically similar, mutually exclusive sets and perform the commonality analysis on the sets. The SMC's used for the commonality analysis are the SMC's for all possible combinations of sets of variables, not of individual regressors.

The commonality table computed in this manner is a summary of the elements of a commonality analysis of the individual regressors. Considering the example in section 3, the four variables may be grouped into two sets, set A containing x_1 and x_2, and set B containing x_3 and x_4. The commonality and uniquenesses of the sets are

$$U_A = U_1 + U_2 + C_{12}$$

$$U_B = U_3 + U_4 + C_{34}$$

$$C_{AB} = C_{13} + C_{14} + C_{23} + C_{24} + C_{123} + C_{124} + C_{134} + C_{234} + C_{1234} .$$

The commonality table is shown in Figure 5.

The algorithm shown in section 7 is also appropriate for sets using the SMCs of all possible sets of variables as input.

Commonality analysis with sets of variables may require matrix inversion with a large number of variables. Some of these variables may be highly correlated with the result that the analysis is subject to possible numerical instability. The user should be careful to make sure the correlations among the independent variables are not extremely high. Numerical experiments by Beaton, Rubin, and Barone (1972) have shown that very high collinearity will affect regression coefficients substantially, but do not affect squared multiple correlations too seriously. Double precision is probably sufficient to avoid most problems of this sort.

	A	B	
U_A	.0298		.0298
U_B		.1447	.1447
C_{AB}	.5080	.5080	.5080
	.5378	.6527	.6825

Figure 5. Commonality table for sets

Creager and Boruch (1969) suggest a combination of factor analysis and regression analysis as an alternative to commonality analysis. Using the notation of this paper, they argue that the matrix $\underset{\sim}{X}$ may be factored as

$$\underset{\sim}{X} = \underset{\sim}{Z}\underset{\sim}{F}$$

where $\underset{\sim}{Z}$ is an orthogonal Nxf matrix of factor scores and $\underset{\sim}{F}$ is an fxm matrix of factor loadings. Using maximum likelihood techniques (see Joreskog, 1967), one can estimate the matrix $\underset{\sim}{F}$ such that

$$\underset{\sim}{F}'\underset{\sim}{Z}'\underset{\sim}{Z}\underset{\sim}{F} = \hat{R}_{xx}$$

where $\hat{R}_{xx}$ is in general an approximation to the correlations among the regressors, and the model becomes (if the regressors were of unit variance)

$$\underset{\sim}{y} = \underset{\sim}{Z}\underset{\sim}{F}\underset{\sim}{\beta} + \underset{\sim}{\varepsilon}$$

$\underset{\sim}{\beta}$ is estimated by regression analysis. Since $Z'Z = I$, the predictable variance $\sigma^2_{\hat{y}}$ is simply the sum of squares of the elements in the vector $\underset{\sim}{F}\underset{\sim}{\beta}$.

If one can hypothesize an underlying factor structure, then the Creager and Boruch method is the natural way to identify the contributions of the factors, and the method may be useful for exploration of found commonalities, but commonality analysis differs in that it simply locates elements of the SMC in the given variables without postulating underlying, orthogonal variables. It would seem that the two methods complement each other and may be used together, commonality analysis perhaps helping develop the factor hypothesis.

6.0 Multivariate Commonality

Multivariate commonality is a technique for assessing the common and unique predictability of several regressors or sets of regressors on a set of $p \geq 1$ regressands. The technique is a simple generalization of univariate commonality and the results of the two will be the same if the value of p is unity. Multivariate commonality is to multivariate analysis very much as univariate commonality is to regression analysis.

One simple method of assessing the commonalities of several regressors on a set of regressands would be to compute a commonality table for each regressand, then sum the several commonality tables into a single multivariate table. Although this might be appropriate for some problems, this procedure might overweight the commonality if two or more of the regressands were very highly correlated with each other and similarly correlated with the regressors. A high commonality for one of the regressands would also show up in the commonality tables of its correlates with the result that the multivariate commonality in the summary table would be very large or, in a sense, counted twice or more. This procedure also varies from common practice in multivariate analysis.

An alternative procedure that avoids redundant predictive power transforms the regressands such that a new set of regressands are computed which have the property that each has a unit variance and is uncorrelated with the other transformed regressands. The transformed variables contain all of the relevant information in the original regressands and can be transformed back to the original if needed. Since the transformed regressands are uncorrelated, the prediction of one is not associated with the prediction of another. The multivariate commonality proposed here can be thought of as performing a

transformation to unit orthogonal regressands, then a univariate commonality analysis on each transformed regressand, and summing the univariate commonality into a multivariate commonality table.

There are many ways to transform the regressands to unit orthogonal orientation. One way is to perform a principal components analysis of the regressands, compute component scores, and then rescale the component scores for unit variance. Alternatively, a Cholesky transformation of the regressands can be performed, and residual scores with unit orthogonal orientation computed. This procedure is equivalent to computing a Gram-Schmid decomposition of the regressands with the result that the first regressand is simply rescaled, the second is taken as a residual from the first, the third as a residual from the first and second, and so forth. Since the Gram-Schmid method is computationally simpler and the commonality tables under either procedure are identical, the Gram-Schmid method is used here.

The actual calculation of multivariate commonalities is only very slightly more complicated and expensive than univariate commonality. The transformed regressands do not need to be computed for each member of the sample. A correlation matrix including both the (untransformed) regressands and the regressors is computed. The correlation matrix of the regressands can be transformed using the MSTD operator which also transforms the correlations of the regressors and the regressands. The calculation of all possible regressions of the regressors on all of the transformed regressands can be computed using the same number of SWPs as in the univariate case. (For a complete discussion of the MSTD and SWP operators, see Beaton (1964).) The SWP operator computes the set of terms of the form $1 - R^2$ where the R^2 are the squared multiple correlations which must be summed. This sum is called

the trace of the explained cross-products matrix. The traces for all combinations of predictors may be converted to multivariate commonalities by the Barone algorithm discussed in the next section.

Multivariate commonality is related to multivariate hypothesis testing in a manner analogous to the relationship of univariate commonality to regular testing. If the Beaton operators are used, the matrix of sums of squares and cross-products of the residuals is computed after each regression. This matrix is called E_s where the subscript s represents the particular regression that was removed. The trace of E_s is the unexplained sum of squares and thus p minus trace (E_s) is the explained sum of squares. The matrix E_s has the property that its latent roots are $1 - \lambda^2$ where λ is the canonical correlation between the regressors in s and all regressands. The determinant of E_s is Wilks' Λ which can be used for testing the hypothesis that there is no predictive power in the regressors in s.

Each of the multivariate uniquenesses can be tested for significance. To test a particular uniqueness, one computes the E_s for that uniqueness, i.e., sweeps all other regressors (or sets of regressors), and computes the matrix E, the residual after all regressors are swept. Wilks' Λ statistic is

$$\Lambda = \frac{\det (E)}{\det (E_s)}$$

The Λ statistic may be used to test the hypothesis that the particular multivariate uniqueness is zero which is equivalent to testing that the regressor(s) add nothing to the canonical correlations.

The traces from which the multivariate commonalities are computed are not bounded by zero and unity as are the squared multiple correlations. If the regressands are completely predictable, then the trace would be p , the number of regressands. However, the traces associated with particular regressors or combinations of regressors may have upper bounds substantially less than p . In fact, the maximum trace for a single regressor is unity. The maximum trace for a set of regressors is the number of regressors or the number of regressands, whichever is smaller. Since the same traces cannot be large, the commonalities may seem disappointingly small. Each commonality should be judged in comparison to its maximum.

7.0 Analysis of Variance

Commonality analysis can be a useful adjunct to non-orthogonal analyses of variance and covariance. The analysis of variance is a procedure to partition the total sum of squares of a "dependent" variable into parts associated with one or more "independent" variables or factors for the purpose of estimating mean squares and testing hypotheses. We consider here only situations in which there are at least two factors. If the experimental design is balanced, then the partitioning of the sum of squares is straightforward and the part associated with each factor is distinct, thus the sum of the parts associated with each factor plus the error sum of squares add to the total sum of squares. A balanced or orthogonal analysis, therefore, does break up the whole into its parts. If the design is non-orthogonal, i.e., not balanced, then parts of the total sum of squares can be attributed to more than one factor with the result that the total sum of squares is not broken up into distinct parts associated with factors or their interaction.

There are a number of ways in which non-orthogonal analyses can be performed. One way is to order the factors a priori and assign the common part of the sum of squares to the first factor in the ordering, thereby assuring the allocation of all squares to some factor or to error.

Another procedure is to treat each factor or interaction separately so that all other factors are fitted before any hypothesis is tested. This procedure is equivalent to treating each hypothesis

test as a test of the significance of regression coefficients. Consider the model

$$M_0: \quad y = X\underline{\beta} + \varepsilon$$

where the matrix X contains dummy variables corresponding to row, column, and interaction effects and the vector β is partitioned accordingly. Using the notation from the previous sections, let X_s be the subset of dummy variables representing a set of effects (e.g. the row effects). The test of the significance of the corresponding parameters β_s is formed by specifying the hypothesis

$$H_s : \beta_s = 0$$

and the alternate model

$$M_s : y = X_{\overline{s}}\beta_{\overline{s}} + \varepsilon$$

since M_s implies H_s is true. The sum of squares due to the β_s is then

$$SS(\beta_s) = y'X_{\overline{s}}(X'_{\overline{s}}X_{\overline{s}})^{-1}X'_{\overline{s}}y - y'X(X'X)^{-1}X'y$$

and the appropriate F statistic is

$$F^{M_s}_{N-n} = \frac{ss(\beta_s)/n_s}{(y'y-y'X(X'X)^{-1}X'y)/(N-n)}$$

Each set of effects can be tested using this procedure although the tests are not strictly independent as is also the case with orthogonal designs. It is convenient to collect the sums of squares, degrees of freedom, mean squares, and F statistics in an analysis of variance table. However, the sums of squares allocated to various factors

plus the error sum of squares do not add up to the total, thus the whole is not completely divided into its parts.

The analysis of variance can also be approached through commonality analysis. The total sum of squares can be partitioned into parts attributable to each set of effects and each combination of sets of effects. These common and unique elements added to the error sum of squares add to the total sum of squares. The tests of the uniquenesses are equivalent to the tests of the sets of effects shown above. However, since there are no hypothesis tests available for common elements, part of the motivation for partitioning variance is lost.

Commonality analysis, therefore, gives new information for non-orthogonal analysis of variance since one can investigate the sums of squares that either did not enter into any hypothesis test or were entered into more than once.

8.0 Algorithms

Commonality analysis requires two computational steps: first, computing all possible SMC's and, then computing the commonality elements. This section discusses the two phases and presents a complete FORTRAN subroutine.

A vector of all possible SMC can be computed using the algorithm of Shatzoff (1968). Shatzoff uses the sweep (SWP) operator (See Beaton (1964) or Dempster (1969)) which adds and deletes variables to a multiple regression equation. The SMC's must be computed in the order shown in/ Figure

The Shatzoff algorithm has the property that each of the $2^m - 1$ sweeps computes a different SMC. The subprogram presented below modifies the method by replenishing the matrix occasionally in order to avoid buildup of computational error.

Although commonality elements can be computed by first forming, then solving simultaneous equations, an algorithm due to Barone is simpler for hand calculation and requires far less computer memory. The algorithm requires as input the SMC's. An additional dummy SMC of value zero as the zeroth element is also needed.

Each element in an m-variable commonality table may be represented as C_s where s is the subscript containing $n_s \leq m$ integers in numerical order and representing the indices of the variables included in C_s. The complementary subscript $\bar{s}$ contains the $n_{\bar{s}}=m-n_s$ indices not included in s. If all m indices are included in s, then $\bar{s}$ is considered to contain no indices.

The Barone algorithm begins the commonality computation of any element C_s by forming an equation with the first element as minus

ommonality Element	G																SMC	LOC
C	0	1	2	12	3	13	23	123	4	14	24	124	34	134	234	1234	r	
C_{1234}	-1	1	1	-1	1	-1	-1	1	1	-1	-1	1	-1	1	1	-1	R_0^2	0
C_{234}		-1		1		1		-1		1		-1		-1		1	R_1^2	1
C_{134}			-1	1			1	-1			1	-1			-1	1	R_{12}^2	2
C_{34}				-1				1				1				-1	R_{12}^2	3
C_{124}					-1	1	1	-1					1	-1	-1	1	R_3^2	4
C_{24}						-1		1						1		-1	R_{13}^2	5
C_{14}							-1	1							1	-1	R_{23}^2	6
$C_4 = U_4$								-1								1	R_{123}^2	7
C_{123}									-1	1	1	-1	1	-1	-1	1	R_4^2	8
C_{23}										-1		1		1		-1	R_{14}^2	9
C_{13}											-1	1			1	-1	R_{24}^2	10
$C_3 = U_3$												-1				1	R_{124}^2	11
C_{12}													-1	1	1	-1	R_{34}^2	12
$C_2 = U_2$														-1		1	R_{134}^2	13
$C_1 = U_1$															-1	1	R_{234}^2	14
																	R_{1234}^2	15

Fig. 7.1. Commonality equations.

$R^2_{y\bar{s}}$, that is, the SMC with complementary subscript and negative sign. Each index in s is then appended one at a time to those already in $\bar{s}$ to form a set of subscripts $\bar{s}'$; each SMC with one of the subscripts in $\bar{s}'$ is added to the equation. The integers in s are then appended to $\bar{s}$ two at a time to form subscripts $\bar{s}'$, and the corresponding SMC are subtracted from the equation. This process continues with more and more indices in s appended $\bar{s}$ and with the sign of the addition changing at each step until all n_s indices in s are used.

For example, let us find the equation for C_{13} in an analysis with four variables $(m = 4)$. The complement of the subscript $s = 13$ is $\bar{s} = 24$, thus the first element is

$$-R^2_{y24} \ .$$

Appending the elements of s one at a time to $\bar{s}$, the subscripts in $\bar{s}' = (124,234)$ and thus the next elements added to C_{13} are

$$+ R^2_{Y124} + R^2_{Y234} \ .$$

Finally, appending the elements of $\bar{s}$ two at a time, we have $\bar{s}'' = (1234)$ and thus next element subtracted from C_{13} is

$$-R^2_{Y1234} \ .$$

The equation for C_{13} is then

$$C_{13} = -R^2_{Y24} + R^2_{Y124} + R^2_{Y234} - R^2_{Y1234} \ .$$

For another example, C_{1234} has a complementary subscript 0 , thus the first element R^2_0 is subtracted

$$-0 \ ,$$

each integer in n is added one at a time,

$$+ R^2_{Y1} + R^2_{Y2} + R^2_{Y3} + R^2_{Y4} \quad ,$$

then the values selected by pairs of integers are subtracted,

$$-R^2_{Y12} -R^2_{Y13} -R^2_{Y14} -R^2_{Y23} -R^2_{Y24} -R^2_{Y34} \quad ,$$

and the 3 index combinations added

$$+ R^2_{Y123} + R^2_{Y124} +R^2_{Y134} + R^2_{Y234} \quad ,$$

and, finally, the SMC with all subscripts is subtracted,

$$-R^2_{Y1234} \quad .$$

The commonality element is

$$C_{1234} = R^2_{Y1} + R^2_{Y2} + R^2_{Y3} + R^2_{Y4} -R^2_{Y12} -R^2_{Y13}$$
$$R^2_{Y14} -R^2_{Y23} -R^2_{Y24} -R^2_{Y34}$$
$$+ R^2_{Y123} + R^2_{Y124} +R^2_{Y34} + R^2_{Y234} -R^2_{Y1234} \quad .$$

The equations for all commonalities for $M = 4$ are shown in Figure 7.1. The left hand column contains the commonality element for a four variable problem and the right hand columns contain the squared multiple correlations and their relative location in a computer program. The main body of the figure contains the matrix G which may be used in the equation

$$\underline{c} = G\underline{r}$$

to compute the commonalities from the vector of SMC's. The matrix G was computed using the Barone algorithm not by matrix inversion, but may be checked by forming a matrix G^{-1} defining the relationship of the c_s and SMC in a matrix, then multiplying GG^{-1} which must result in an identity matrix.

The reader may verify the matrix G and the commonality table in Figure 3.2 by multiplying G by the vector $\underline{r}$ in Figure 4.-1.

A computer subroutine for computing commonalities is shown in Figure 7.2. The subroutine accepts as input a cross-products or correlation matrix, computes all possible SMC, and then a commonality table. Several regressands and sets of regressors may be used. The program does not compute multivariate commonality.

The computer program uses the binary nature of computers to advantage. The presence or absence of a subscript is coded by a zero or one bit in the appropriate position of a memory register. The binary word is evaluated as the location in SMC table of the appropriate element. For example, the complement of C_{24} os

$$\text{comp } (C_{24}) = 0101_2 = \text{LOC}(5) = R^2_{Y13} \ .$$

The variables position are read from right to left, thus 0101_2 indicates the presence of variables X_1 and X_3.

The calling sequence and definition of parameters is shown in Figure 7.2.

CALLING SEQUENCE

CALL COMMON (C,MPI,LC,NC,LH,NH,IK,NK,TITL,LD,ND,HDS,HACR)

PARAMETERS:

C (MPI,MPI)	:	INPUT CROSS-PRODUCTS MATRIX
MPI	:	DIMENSION OF C
LC (NC)	:	INTEGER VECTOR CONTAINING THE ROW (COLUMN) NUMBERS OF THE CONCOMITANT VARIABLES.
NC	:	NUMBER OF CONCOMITANT VARIABLES DEFINED IN 'LC'.
LH (NH)	:	INTEGER VECTOR CONTAINING THE ROW (COLUMN) NUMBERS OF THE INDEPENDENT VARIABLES.
NH	:	THE NUMBER OF INDEPENDENT VARIABLES DEFINED IN 'LH'.
IK (NK)	:	INTEGER VECTOR CONTAINING THE NUMBER OF INDEPENDENT VARIABLES TO BE GROUPED FROM THE 'LH' LIST TO FORM EACH OF THE NK 'SETS' USED IN THE COMMONALITY ANALYSIS. THE SUM OF THE ELEMENTS IN THIS VECTOR MUST EQUAL NH.
NK	:	NUMBER OF SETS TO BE FORMED USING THE 'IK' LIST.
TITL	:	ANY 'BCD' HEADING OF LESS THAN 120 CHARACTERS. THIS WILL BE PRINTED PRECEEDING EACH COMMON-ABILITY TABLE.
LD	:	INTEGER VECTOR CONTAINING THE ROW (COLUMN) NUMBERS OF THE DEPENDENT VARIABLES. A SEPARATE COMMONALITY TABLE WILL BE COMPUTED FOR EACH DEPENDENT VARIABLE.
ND	:	NUMBER OF DEPENDENT VARIABLES
HDS (ND)	:	THIS VECTOR MUST CONTAIN THE ALPHANUMERIC TITLE OF EACH DEPENDENT VARIABLE. THE TITLES MUST CORRESPOND TO THE VARIABLES DEFINED IN THE 'LD' LIST OF DEPENDENT VARIABLES.

Fig. 7.2. Univariate commonality program.

HACR (NK) : THIS VECTOR MUST CONTAIN THE ALPHANUMERIC TITLES OF EACH OF THE 'NK' SETS DEFINED BY THE 'LH', 'IK' LISTS.

CALLED ROUTINES
SWPSET,NEWPG

EXAMPLE

GIVEN: 'C' IS A 10 x 10 CROSS PRODUCTS MATRIX.
THERE ARE 2 DEPENDENT VARIABLES LOCATED IN POSITIONS 1 and 2 IN THE MATRIX.
THERE ARE 8 INDEPENDENT VARIABLES LOCATED IN POSITIONS 3 thru 9.
POSITION 10 CONTAINS THE VARIABLES CORRESPONDING TO THE OVERALL MEAN.

IN THIS EXAMPLE WE WANT TO ACCOMPLISH THE FOLLOWING FOR EACH OF THE 2 DEPENDENT VARIABLES

1) ADJUST THE CROSS PRODUCTS MATRIX BY REMOVING THE GRAND MEAN

2) FORM 3 'SETS' FROM THE INDEPENDENT VARIABLES. THE FIRST SET CONSISTING OF VARIABLES (3, 4 and 6), THE SECOND SET OF VARIABLES (5, 7) AND THE THIRD SET OF VARIABLES (6, 8, 9).

3) PERFORM A COMMONALITY ANALYSIS USING THESE 3 SETS OF INDEPENDENT VARIABLES.

THE FOLLOWING FORTRAN STATEMENTS WILL ACCOMPLISH THE REQUIRED ANALYSIS.

```
DIMENSION C(10,10),LH(8),LC(1),IK(3),LD(2),HDS(2),HACR(3)

DATA  LH/3,4,6,5,7,6,8,9/ ,NH/8/
DATA  IK/  3,   2,   3, / ,NK/3/
DATA  LC/10/,NC/1/
DATA  LD/1,2/,ND/2/
DATA  HDS/6HDEP.  1,6HDEP.  2 /
DATA  HACR/6HSET  1 ,6HSET  2 , 6HSET 3 /

CALL COMMON(C,10,LC,NC,LH,NH,IK,NK,12H  ANY HEADING, LD,ND,HDS,
1       HACR)
```

Fig. 7.2 (Cont'd). Univariate commonality program.

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